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MATHEMATICAL MODEL OF DYNAMIC HEAT- AND MASS-EXCHANGE PROCESSES IN A GLASSMAKING FURNACE

V. É. Dzhashitov, V. M. Pankratov, and A. V. Golikov

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Theoretical foundations have been developed and a mathematical model of dynamic heat- and mass-exchange processes in a glassmaking furnace has been constructed, which makes it possible to calculate and visualize nonstationary temperature fields in the furnace volume and nonstationary fields of velocities of convective glass-mass flow in the molten bath. Test calculations for a specific structure of a glassmaking furnace which have shown the effectiveness of the model constructed have been performed.

Formulation of the Problem and Theoretical Foundations of Its Solution. The necessity of designing and creating modern glasses with prescribed properties calls for knowledge of the character and parameters of convective flows and thermal processes occurring in glassmaking furnaces. On the other hand, the complexity of these heat-exchange processes [1, 2] is responsible for the wide range of theoretical and experimental methods and means employed in studying them.

The present work seeks to construct and investigate a mathematical model of coupled nonstationary heat- and mass-exchange processes occurring in a glassmaking furnace (Fig. 1).

We pose the problem of obtaining the equations of heat- and mass-exchange processes occurring in the molten bath and other zones of the glassmaking furnace and of determining and visualizing nonuniform, three-dimensional, nonstationary temperature fields and fields of velocities of glass-mass flow.

The experience gained by the authors in studying heat- and mass-exchange processes in complex devices and systems of air-space instrument engineering [3] makes it possible to realize the idea of double application of modern methods and means of investigation to the problem formulated.

In the molten bath of a standard continuous glassmaking furnace (Fig. 1a, b, and f), there are zones of maximum (region of the third burner in the melting zone) and minimum temperatures (regions of the first and sixth burners in the melting zone and production regions in the cooling zone) creating vertical and horizontal temperature gradients and causing the main thermal convection in the longitudinal direction of the furnace. There are also additional convective flows in the transverse direction due to the corresponding temperature gradients.

The basic initial equations [4–6] are as follows:

(1) the coupled nonstationary equation of nonisothermal motion of an incompressible viscous continuum in the Overbeck–Boussinesq approximation

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\,\mathbf{V} = \mathbf{g}\beta T - \frac{1}{\rho}\,\overline{\text{grad}}\,P + \nu\nabla^2\mathbf{V}\,; \tag{1}$$

(2) the energy (heat- and mass-transfer) equation

$$\frac{\partial T}{\partial t} + \mathbf{V}\,\overline{\text{grad}}\,T = k\nabla^2 T\,; \tag{2}$$

(3) the continuity equation

Institute of Problems of Fine Mechanics and Control, Russian Academy of Sciences, 24 Rabochaya Str., Saratov, 410028, Russia; email: iptmu@forpost.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 5, pp. 117–124, September–October, 2003. Original article submitted December 20, 2002; revision submitted April 14, 2003.

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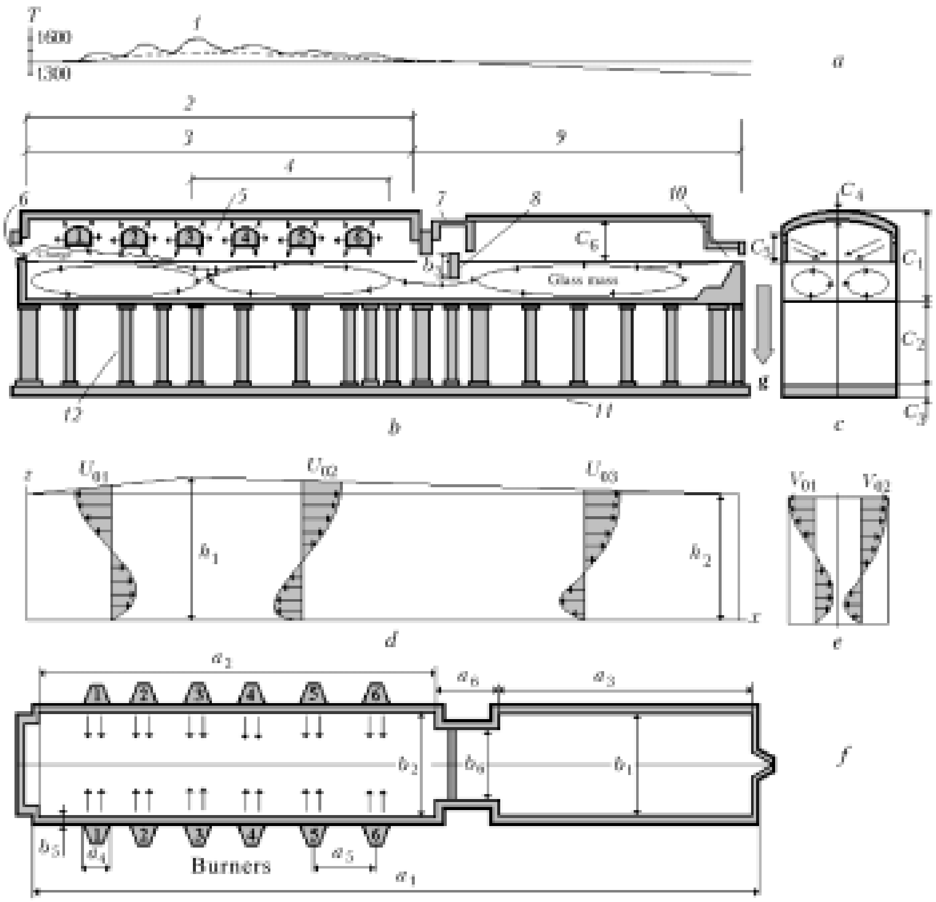

Fig. 1. Glassmaking furnace, diagram of heat exchange and of motion of glass-mass flows (C_1, C_2, ..., C_6, b_1, b_2, ..., b_6, a_1, a_2, ..., a_6 are the geometric dimensions): a) standard temperature curve with a temperature rise 1 at the burner flames; b) longitudinal section of the furnace [2) melting tank, 3) melting zone, 4) refining zone, 5) gas-medium zone, 6) charging zone, 7) knuckle, 8) water refrigerator, 9) cooling zone, 10) production zone, 11) bed plate, 12) supporting columns, figures 1, 2, ..., 6 inside the furnace are the numbers of the burners]; c) cross section of the furnace; d) flow-velocity profiles in the longitudinal section (U_{01}, U_{02}, and U_{03}, are the maximum horizontal components of the velocities of glass-mass flow in the zones of melting, refining, and cooling respectively; h_1 and h_2 are the characteristic heights of the glass mass); e) flow-velocity profiles in the cross section (V_{01} and V_{02} are the maximum horizontal components of the velocities of glass-mass flow in the left-hand and right-hand zones); f) top view of the glassmaking furnace (figures 1, 2, ..., 6 inside the furnace are the numbers of the burners).

$$\operatorname{div} \mathbf{V} = 0 . \tag{3}$$

The coupled problem described by the system of equations (1)–(3) is solved with allowance for the vertical temperature gradients occurring in the molten bath of the furnace.

According to [5, 6], it is assumed that the system possesses translational invariance in the coordinate y; therefore, the variables in Eqs. (1)–(3) depend on two spatial coordinates: the height z and the horizontal coordinate x, which is perpendicular to the axis of the convective flows (Fig. 1d).

Let the following representations hold:

$$\mathbf{V} = \mathbf{V}\,(u\,(x, z, t)\,,\, w\,(x, z, t))\,, \tag{4}$$

$$T\,(x, z, t) = T_0 + \Delta T - \frac{\Delta T}{h}\, z + \theta\,(x, z, t)\,. \tag{5}$$

Introducing the stream function $\psi(x, z, t)$ such that

$$u = -\frac{\partial \psi}{\partial z}\,, \quad w = \frac{\partial \psi}{\partial x}\,, \tag{6}$$

we obtain the automatic fulfillment of the continuity equation (3).

Applying the rotor operator to Eq. (1) to eliminate pressure and allowing for representation (5) for the temperature fields, from (1) and (2) we have the equations written in terms of the stream function $\psi(x, z, t)$ and the temperature deviation $\theta(x, z, t)$:

$$\frac{\partial}{\partial t}(\nabla^2 \psi) = -\frac{\partial \psi}{\partial x}\frac{\partial}{\partial z}(\nabla^2 \psi) + \frac{\partial \psi}{\partial z}\frac{\partial}{\partial x}(\nabla^2 \psi) + \nu \nabla^4 \psi + g\beta \frac{\partial \theta}{\partial x}\,, \tag{7}$$

$$\frac{\partial \theta}{\partial t} = -\frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial z} + \frac{\partial \psi}{\partial z}\frac{\partial \theta}{\partial x} + \frac{\Delta T}{h}\frac{\partial \psi}{\partial z} + k \nabla^2 \theta\,. \tag{8}$$

We transform (7) and (8) to a system of ordinary differential equations based on the Galerkin method [5, 6]. For this purpose we introduce the following representations of the functions $\psi(x, z, t)$ and $\theta(x, z, t)$:

$$\psi\,(x, z, t) = \psi_1\,(t)\,\sin\frac{\pi x}{\lambda}\,\sin\frac{\pi z}{h}\,, \tag{9}$$

$$\theta\,(x, z, t) = \theta_1\,(t)\,\cos\frac{\pi x}{\lambda}\,\sin\frac{\pi z}{h} - \theta_2\,(t)\,\sin\frac{2\pi z}{h}\,, \tag{10}$$

$$u = -\frac{\pi}{h}\,\psi_1\,(t)\,\sin\frac{\pi x}{\lambda}\,\cos\frac{\pi z}{h}\,, \quad w = \frac{\pi}{\lambda}\,\psi_1\,(t)\,\cos\frac{\pi x}{\lambda}\,\sin\frac{\pi z}{h}\,. \tag{11}$$

In accordance with the Galerkin method, functions (9)–(11) exactly satisfy the boundary conditions

$$w\big|_{z=0,h} = 0\,, \quad \frac{\partial u}{\partial z}\bigg|_{z=0,h} = 0\,, \quad \theta\big|_{z=0,h} = 0\,, \quad \psi\big|_{z=0,h} = 0\,. \tag{12}$$

The impermeability of the bath walls and the presence of the free glass-mass surface are taken into account (tangential stresses are absent for $z = h$).

Now, according to the Galerkin method, by substituting (9) and (10) into (7) and (8) and disregarding the harmonics of third order or higher, after transformations we obtain the following system of nonlinear ordinary differential equations:

$$\dot{\psi}_1 = -\frac{\nu \pi^2\,(\lambda^2 + h^2)}{\lambda^2 h^2}\,\psi_1 + \frac{g\beta\lambda h^2}{\pi\,(\lambda^2 + h^2)}\,\theta_1\,, \tag{13}$$

$$\dot{\theta}_1 = -\frac{\pi^2}{\lambda h}\,\psi_1 \theta_2 + \Delta T\,\frac{\pi}{\lambda h}\,\psi_1 - \frac{k\pi^2\,(\lambda^2 + h^2)}{\lambda^2 h^2}\,\theta_1\,, \tag{14}$$

$$\dot{\theta}_2 = \frac{\pi^2}{2\lambda h} \psi_1 \theta_1 - k \frac{4\pi^2}{h^2} \theta_2 . \qquad (15)$$

The uncoupled problem [3, 4] described by the equations which represent a particular case of system (1)–(3) without nonlinear terms is solved with allowance for the horizontal temperature gradients. It is assumed that the configuration of the temperature field, which has the horizontal components, is known and prescribed.

Under the assumptions made, the solving system of equations and relations in the polar coordinate system will take the following form:

(1) the equation of nonisothermal motion of a viscous continuum

$$\frac{\partial^2 V_\varphi (z, \varphi)}{\partial z^2} = \frac{1}{\mu R} \frac{\partial P (\varphi)}{\partial \varphi} - \frac{F_\varphi (\varphi)}{\nu} ; \qquad (16)$$

(2) the projection of mass forces

$$F_\varphi = - g \left[1 - \beta \Delta T (\varphi)\right] \cos \varphi ; \qquad (17)$$

(3) the configuration of the temperature field

$$\Delta T (\varphi) = \Delta T_0 \cos \varphi ; \qquad (18)$$

(4) the boundary conditions

$$V_\varphi (0, \varphi) = V_\varphi (h_{\text{ann}}, \varphi) = 0 . \qquad (19)$$

Integrating (16) with respect to z with account for (17) and (18) and boundary conditions (19), we have the following expression for the velocity component:

$$V_\varphi (z, \varphi) = \left(\frac{1}{\mu R} \frac{dP}{d\varphi} - \frac{F_\varphi}{\nu} \right) z (z - h_{\text{ann}}) / 2 . \qquad (20)$$

Averaging (20) from 0 to 2π over φ and from 0 to h_{ann} over z and taking into account that $P(2\pi) - P(0) = 0$, we obtain the formula

$$\langle V_\varphi \rangle = \frac{g \beta \Delta T_0}{\nu} \frac{h_{\text{ann}}^2}{12} . \qquad (21)$$

As we see, the flow-velocity component due to the horizontal temperature gradients is in direct proportion to the coefficient of thermal expansion of the glass mass and the temperature difference and in inverse proportion to the viscosity of the glass mass. To apply the relation obtained to evaluation of the convection in the molten bath of the glassmaking furnace we introduce the weight factor $p \sim h_{\text{ann}}^2/12$. Varying this coefficient, we can select the prescribed component of the flow velocity, which is due to the horizontal temperature gradients.

Thus, for evaluation of the maximum components of the velocities of glass-mass flow caused by the horizontal temperature gradients in the gravitational field with allowance for the continuity of the flow we can use the following relations:

$$\langle V_\varphi \rangle = \frac{g \beta \Delta T_0}{\nu} p , \quad \langle W_\varphi \rangle = \frac{g \beta \Delta T_0 h}{\nu \lambda} p . \qquad (22)$$

The values of the velocities determined by formulas (22) are involved as additional terms in the representations of the velocity fields (11):

$$u = \left(-\frac{\pi}{h} \psi_1(t) \mp \langle V_\varphi \rangle \right) \sin \frac{\pi x}{\lambda} \sin \frac{\pi z}{h}, \quad w = \left(\frac{\pi}{\lambda} \psi_1(t) \pm \langle W_\varphi \rangle \right) \cos \frac{\pi x}{\lambda} \sin \frac{\pi z}{h}. \tag{23}$$

The nonlinear equations and relations (4)–(23) obtained represent a basic theoretical foundation for solution of the formulated problem of construction of a mathematical model of heat- and mass-exchange processes in a glassmaking furnace.

Mathematical Model. For a comprehensive analysis of the dynamic thermal processes in a glassmaking furnace we have constructed a mathematical model described by a system of differential equations and analytical relations which enable us to allow for heat and mass transfer by conduction, free and forced convection, and radiation.

For description of the heat- and mass-exchange processes in the glass mass filling the molten bath of the glassmaking furnace we employ the differential equations and relations obtained of the form (4)–(23). For description of the heat- and mass-exchange processes in other zones of the furnace we use the differential equations of a modified elementary-balance method [3].

The external heat-release sources, heat exchange with the ambient medium, operation of the system of automatic temperature control, thermophysical and geometric characteristics of the structural elements of the furnace, the glass mass, the charged mixture, and the slag (density, heat capacity, thermal conductivity, viscosity, emissivity of the surfaces, and others) are allowed for on the right-hand sides and in the coefficients of the equations.

We have also developed the computation algorithms and the algorithms of transition from the initial system of partial equations to a system of nonlinear ordinary differential equations describing the considered heat-exchange processes in the glassmaking furnace with allowance for the basic relations characteristic of the processes of glassmaking [1–3].

The complete system of differential equations of the mathematical model of heat- and mass-exchange processes in the longitudinal section of the glassmaking furnace has the following form:

the burners and the thermal-control system

$$\begin{aligned}
&c_1\dot{T}_1 + q_{1,2}(T_1 - T_2) + p_1 q_{1,7}(T_1 - T_7) + p_2 q_{1,13}(T_1 - T_{13}) = N_1, \\
&c_2\dot{T}_2 + q_{1,2}(T_2 - T_1) + p_2 q_{2,14}(T_2 - T_{14}) + q_{2,3}(T_2 - T_3) + p_1 q_{2,8}(T_2 - T_8) = N_2, \\
&c_3\dot{T}_3 + q_{2,3}(T_3 - T_2) + p_2 q_{3,15}(T_3 - T_{15}) + q_{3,4}(T_3 - T_4) + p_1 q_{3,9}(T_3 - T_9) = N_3, \\
&c_4\dot{T}_4 + q_{3,4}(T_4 - T_3) + p_2 q_{4,16}(T_4 - T_{16}) + q_{4,5}(T_4 - T_5) + p_1 q_{4,10}(T_4 - T_{10}) = N_4, \\
&c_5\dot{T}_5 + q_{4,5}(T_5 - T_4) + p_2 q_{5,17}(T_5 - T_{17}) + q_{5,6}(T_5 - T_6) + p_1 q_{5,11}(T_5 - T_{11}) = N_5, \\
&c_6\dot{T}_6 + q_{5,6}(T_6 - T_5) + p_2 q_{6,18}(T_6 - T_{18}) + p_1 q_{6,12}(T_6 - T_{12}) = N_6;
\end{aligned} \tag{24}$$

the arch and lateral walls in the melting zone

$$\begin{aligned}
&c_7\dot{T}_7 + p_1 q_{1,7}(T_7 - T_1) + q_{7,8}(T_7 - T_8) + q_{7\text{med}}(T_7 - T_{\text{med}1}) = 0, \\
&c_8\dot{T}_8 + q_{7,8}(T_8 - T_7) + p_1 q_{2,8}(T_8 - T_2) + q_{8,9}(T_8 - T_9) + q_{8\text{med}}(T_8 - T_{\text{med}2}) = 0, \\
&c_9\dot{T}_9 + q_{8,9}(T_9 - T_8) + p_1 q_{3,9}(T_9 - T_3) + q_{9,10}(T_9 - T_{10}) + q_{9\text{med}}(T_9 - T_{\text{med}3}) = 0, \\
&c_{10}\dot{T}_{10} + q_{9,10}(T_{10} - T_9) + p_1 q_{4,10}(T_{10} - T_4) + q_{10,11}(T_{10} - T_{11}) + q_{10\text{med}}(T_{10} - T_{\text{med}4}) = 0, \\
&c_{11}\dot{T}_{11} + q_{10,11}(T_{11} - T_{10}) + p_1 q_{5,11}(T_{11} - T_5) + q_{11,12}(T_{11} - T_{12}) + q_{11\text{med}}(T_{11} - T_{\text{med}5}) = 0, \\
&c_{12}\dot{T}_{12} + q_{11,12}(T_{12} - T_{11}) + p_1 q_{6,12}(T_{12} - T_6) + q_{12\text{med}}(T_{12} - T_{\text{med}6}) + q_{12,29}(T_{12} - T_{29}) = 0;
\end{aligned} \tag{25}$$

the mixture and the glass mass in the surface layer in the melting zone

$$c_{13}\dot{T}_{13} + q_{13,14}(T_{13} - T_{14}) + p_2 q_{1,13}(T_{13} - T_1) + q_{13\text{med}}(T_{13} - T_{\text{med1}}) = 0 ,$$

$$c_{14}\dot{T}_{14} + q_{13,14}(T_{14} - T_{13}) + q_{14,15}(T_{14} - T_{15}) + p_2 q_{2,14}(T_{14} - T_2) + q_{14\text{med}}(T_{14} - T_{\text{med2}}) = 0 ,$$

$$c_{15}\dot{T}_{15} + q_{14,15}(T_{15} - T_{14}) + q_{15,16}(T_{15} - T_{16}) + p_2 q_{3,15}(T_{15} - T_3) + q_{15\text{med}}(T_{15} - T_{\text{med3}}) = 0 ,$$

$$c_{16}\dot{T}_{16} + q_{15,16}(T_{16} - T_{15}) + q_{16,17}(T_{16} - T_{17}) + p_2 q_{4,16}(T_{16} - T_4) + q_{16\text{med}}(T_{16} - T_{\text{med4}}) = 0 , \tag{26}$$

$$c_{17}\dot{T}_{17} + q_{16,17}(T_{17} - T_{16}) + q_{17,18}(T_{17} - T_{18}) + p_2 q_{5,17}(T_{17} - T_5) + q_{17\text{med}}(T_{17} - T_{\text{med5}}) = 0 ,$$

$$c_{18}\dot{T}_{18} + q_{17,18}(T_{18} - T_{17}) + p_2 q_{6,18}(T_{18} - T_6) + q_{18\text{med}}(T_{18} - T_{\text{med6}}) + q_{18,26}(T_{18} - T_{26}) = 0 ;$$

the mixture and the glass mass in the surface layer in the cooling zone

$$c_{26}\dot{T}_{26} + q_{18,26}(T_{26} - T_{18}) + q_{26,27}(T_{26} - T_{27}) + p_1 q_{26,29}(T_{26} - T_{29}) + q_{26\text{med}}(T_{26} - T_{\text{med7}}) = 0 ,$$

$$c_{27}\dot{T}_{27} + q_{26,27}(T_{27} - T_{26}) + q_{27,28}(T_{27} - T_{28}) + p_1 q_{27,30}(T_{27} - T_{30}) + q_{27\text{med}}(T_{27} - T_{\text{med8}}) = 0 , \tag{27}$$

$$c_{28}\dot{T}_{28} + q_{27,28}(T_{28} - T_{27}) + p_1 q_{28,31}(T_{28} - T_{31}) + q_{28\text{med}}(T_{28} - T_{\text{med9}}) = 0 ;$$

the arch and lateral walls in the cooling zone

$$c_{29}\dot{T}_{29} + q_{12,29}(T_{29} - T_{12}) + p_1 q_{26,29}(T_{29} - T_{26}) + q_{29,30}(T_{29} - T_{30}) + q_{29\text{med}}(T_{29} - T_{\text{med7}}) = 0 ,$$

$$c_{30}\dot{T}_{30} + q_{29,30}(T_{30} - T_{29}) + p_1 q_{27,30}(T_{30} - T_{27}) + q_{30,31}(T_{30} - T_{31}) + q_{30\text{med}}(T_{30} - T_{\text{med8}}) = 0 , \tag{28}$$

$$c_{31}\dot{T}_{31} + q_{30,31}(T_{31} - T_{30}) + p_1 q_{28,31}(T_{31} - T_{28}) + q_{31\text{med}}(T_{31} - T_{\text{med9}}) = 0 ;$$

the equation of motion of the glass mass in the melting zone (the first to third burners)

$$\dot{\psi}_{1\text{melt}} = -\frac{\nu_{\text{melt}}\pi^2(\lambda_{\text{melt}}^2 + h_{\text{melt}}^2)}{\lambda_{\text{melt}}^2 h_{\text{melt}}^2}\psi_{1\text{melt}} + \frac{g\beta\lambda_{\text{melt}}h_{\text{melt}}^2}{\pi(\lambda_{\text{melt}}^2 + h_{\text{melt}}^2)}\theta_{1\text{melt}} ,$$

$$\dot{\theta}_{1\text{melt}} = -\frac{\pi^2}{\lambda_{\text{melt}}h_{\text{melt}}}\psi_{1\text{melt}}\theta_{2\text{melt}} + \frac{(T_{15} - T_{13})\pi}{\lambda_{\text{melt}}h_{\text{melt}}}\psi_{1\text{melt}} - \frac{k_{\text{melt}}\pi^2(\lambda_{\text{melt}}^2 + h_{\text{melt}}^2)}{\lambda_{\text{melt}}^2 h_{\text{melt}}^2}\theta_{1\text{melt}} , \tag{29}$$

$$\dot{\theta}_{2\text{melt}} = \frac{\pi^2}{2\lambda_{\text{melt}}h_{\text{melt}}}\psi_{1\text{melt}}\theta_{1\text{melt}} - k_{\text{melt}}\frac{4\pi^2}{h_{\text{melt}}^2}\theta_{2\text{melt}} ;$$

the fields of temperatures and velocities of the glass mass in the melting zone (the first to third burners)

$$T_{\text{melt}}(x, z, t) = (T_{13} + T_{14} + T_{15})/3 + \theta_{\text{melt}}(x, z, t) ,$$

$$\psi_{\text{melt}}(x, z, t) = \psi_{1\text{melt}}(t)\sin\frac{\pi x}{\lambda_{\text{melt}}}\sin\frac{\pi z}{h_{\text{melt}}} ,$$

$$\theta_{\text{melt}}(x, z, t) = \theta_{1\text{melt}}(t)\cos\frac{\pi x}{\lambda_{\text{melt}}}\sin\frac{\pi z}{h_{\text{melt}}} - \theta_{2\text{melt}}(t)\sin\frac{2\pi z}{h_{\text{melt}}} ;$$

$$u_{melt} = -\frac{\pi}{h_{melt}} \psi_{1melt}(t) \sin\frac{\pi x}{\lambda_{melt}} \cos\frac{\pi z}{h_{melt}} - p_9 \frac{g\beta}{\nu_{melt}} (T_{15} - T_{13}) \sin\frac{\pi x}{\lambda_{melt}} \cos\frac{\pi z}{h_{melt}} , \quad (30)$$

$$w_{melt} = \frac{\pi}{\lambda_{melt}} \psi_{1melt}(t) \cos\frac{\pi x}{\lambda_{melt}} \sin\frac{\pi z}{h_{melt}} + p_9 \frac{h_{melt}}{\lambda_{melt}} \frac{g\beta}{\nu_{melt}} (T_{15} - T_{13}) \cos\frac{\pi x}{\lambda_{melt}} \sin\frac{\pi z}{h_{melt}} ;$$

the equations of motion of the glass mass in the refining zone (the fourth to sixth burners)

$$\dot{\psi}_{1ref} = -\frac{\nu_{ref}\pi^2 (\lambda_{ref}^2 + h_{ref}^2)}{\lambda_{ref}^2 h_{ref}^2} \psi_{1ref} + \frac{g\beta\lambda_{ref} h_{ref}^2}{\pi (\lambda_{ref}^2 + h_{ref}^2)} \theta_{1ref} ,$$

$$\dot{\theta}_{ref} = -\frac{\pi^2}{\lambda_{ref} h_{ref}} \psi_{1ref}\theta_{2ref} + \frac{(T_{16} - T_{18})\pi}{\lambda_{ref} h_{ref}} \psi_{1ref} - \frac{k_{ref}\pi^2 (\lambda_{ref}^2 + h_{ref}^2)}{\lambda_{ref}^2 h_{ref}^2} \theta_{1ref} , \quad (31)$$

$$\dot{\theta}_{2ref} = \frac{\pi^2}{2\lambda_{ref} h_{ref}} \psi_{1ref}\theta_{1ref} - k_{ref} \frac{4\pi^2}{h_{ref}^2} \theta_{2ref} ;$$

the fields of temperatures and velocities of the glass mass in the refining zone (the fourth to sixth burners)

$$T_{ref}(x, z, t) = (T_{16} + T_{17} + T_{18})/3 + \theta_{ref}(x, z, t) ,$$

$$\psi_{ref}(x, z, t) = \psi_{1ref}(t) \sin\frac{\pi x}{\lambda_{ref}} \sin\frac{\pi z}{h_{ref}} ,$$

$$\theta_{ref}(x, z, t) = \theta_{1ref}(t) \cos\frac{\pi x}{\lambda_{ref}} \sin\frac{\pi z}{h_{ref}} - \theta_{2ref}(t) \sin\frac{2\pi z}{h_{ref}} , \quad (32)$$

$$u_{ref} = -\frac{\pi}{h_{ref}} \psi_{1ref}(t) \sin\frac{\pi x}{\lambda_{ref}} \cos\frac{\pi z}{h_{ref}} + p_9 \frac{g\beta}{\nu_{ref}} (T_{16} - T_{18}) \sin\frac{\pi x}{\lambda_{ref}} \cos\frac{\pi z}{h_{ref}} ,$$

$$w_{ref} = \frac{\pi}{\lambda_{ref}} \psi_{1ref}(t) \cos\frac{\pi x}{\lambda_{ref}} \sin\frac{\pi z}{h_{ref}} - p_9 \frac{h_{ref}}{\lambda_{ref}} \frac{g\beta}{\nu_{ref}} (T_{16} - T_{18}) \cos\frac{\pi x}{\lambda_{ref}} \sin\frac{\pi z}{h_{ref}} ;$$

the equations of motion of the glass mass in the cooling zone

$$\dot{\psi}_{1cool} = -\frac{\nu_{cool}\pi^2 (\lambda_{cool}^2 + h_{cool}^2)}{\lambda_{cool}^2 h_{cool}^2} \psi_{1cool} + \frac{g\beta\lambda_{cool} h_{cool}^2}{\pi (\lambda_{cool}^2 + h^2)} \theta_{1cool} ,$$

$$\dot{\theta}_{1cool} = -\frac{\pi^2}{\lambda_{cool} h_{cool}} \psi_{1cool}\theta_{2cool} + \frac{(T_{26} - T_{28})\pi}{\lambda_{cool} h_{cool}} \psi_{1cool} - \frac{k_{cool}\pi^2 (\lambda_{cool}^2 + h_{cool}^2)}{\lambda_{cool}^2 h_{cool}^2} \theta_{1cool} , \quad (33)$$

$$\dot{\theta}_{2cool} = \frac{\pi^2}{2\lambda_{cool} h_{cool}} \psi_{1cool}\theta_{1cool} - k_{cool} \frac{4\pi^2}{h_{cool}^2} \theta_{2cool} ;$$

the fields of temperatures and velocities of the glass mass in the cooling zone

$$T_{cool}(x, z, t) = (T_{26} + T_{27} + T_{28})/3 + \theta_{cool}(x, z, t) ,$$

$$\psi_{\text{cool}}(x, z, t) = \psi_{1\text{cool}}(t) \sin \frac{\pi x}{\lambda_{\text{cool}}} \sin \frac{\pi z}{h_{\text{cool}}} ,$$

$$\theta_{\text{cool}}(x, z, t) = \theta_{1\text{cool}}(t) \cos \frac{\pi x}{\lambda_{\text{cool}}} \sin \frac{\pi z}{h_{\text{cool}}} - \theta_{2\text{cool}}(t) \sin \frac{2\pi z}{h_{\text{cool}}} ,$$

$$u_{\text{cool}} = - \frac{\pi}{h_{\text{cool}}} \psi_{1\text{cool}}(t) \sin \frac{\pi x}{\lambda_{\text{cool}}} \cos \frac{\pi z}{h_{\text{cool}}} + p_9 \frac{g\beta}{\nu_{\text{cool}}} (T_{26} - T_{28}) \sin \frac{\pi x}{\lambda_{\text{cool}}} \cos \frac{\pi z}{h_{\text{cool}}} , \quad (34)$$

$$w_{\text{cool}} = \frac{\pi}{\lambda_{\text{cool}}} \psi_{1\text{cool}}(t) \cos \frac{\pi x}{\lambda_{\text{cool}}} \sin \frac{\pi z}{h_{\text{cool}}} - p_9 \frac{h_{\text{cool}}}{\lambda_{\text{cool}}} \frac{g\beta}{\nu_{\text{cool}}} (T_{26} - T_{28}) \cos \frac{\pi x}{\lambda_{\text{cool}}} \sin \frac{\pi z}{h_{\text{cool}}} .$$

The law of control of the temperature in the *i*th gas burner is

$$N_i = \begin{cases} N_i^{\max} & \text{at} \quad T_i^{\text{pr}} - T_i \geq T_i^{\text{lin}} , \\ N_i^{\max} (T_i^{\text{pr}} - T_i)/T_i^{\text{lin}} & \text{at} \quad 0 \leq T_i^{\text{pr}} - T_i \leq T_i^{\text{lin}} , \\ 0 & \text{at} \quad T_i^{\text{pr}} - T_i \leq 0 . \end{cases} \quad (35)$$

Analogous equations have also been derived for the cross section of the glassmaking furnace.

The systems of equations and relations obtained are nonlinear and they are solved with standard methods (for example, the Runge–Kutta method).

Characteristics of the Software System and Results of Mathematical Modeling and Their Visualization. The set-up base of the initial data for calculation of the heat- and mass-exchange processes in a glassmaking furnace includes the following basic modules: calculation parameters, thermophysical parameters, geometric parameters, and correction factors.

From the initial database, we calculate the coefficients of the model that allow for the conductive, convective, and radiant heat exchange in the furnace in accordance with [3].

The input data of the mathematical model constructed are the structural parameters of the glassmaking furnace, the thermophysical characteristics of materials and media, temperature disturbing and controlling factors, and other characteristics.

The output data are the three-dimensional, nonuniform, nonstationary temperature fields and fields of velocities of glass-mass flow in the functional zones of the glassmaking furnace.

The total number of calculation points of the mathematical model at which one determines the temperature field in the volume of the glassmaking furnace and the field of flow velocities of the glass mass in the molten bath is ≈5000; about ≈700 parameters are visualized and put out to the protocol at each instant of time with a prescribed discreteness.

In the mathematical model developed and in the supporting TKSTEKLO software system, provision is made for variation of the characteristics of melting regimes and the parameters of the basic structural scheme of the glassmaking furnace.

The TKSTEKLO software system for calculation and visualization of thermal processes in a glassmaking furnace has been realized in two versions in the FORTRAN and C++ programming languages to ensure the maximum speed of calculation and clear visualization of the stationary and nonstationary output data.

The total volume of disk space occupied by the software system is 3.2 Mb. The requirements imposed on the computer are as follows: presence of MS DOS and Windows 95 (or higher) operating systems, no less than 8 Mb of RAM, and monitor resolution no less than 800 × 600 pixels. The calculation time of the nonstationary processes of heat exchange on Pentium-III and -IV computers is 800 to 1500 times shorter than the real duration of the processes, which enables us to solve not only the analysis problems but also the synthesis problems on optimization of the furnace parameters and the control actions.

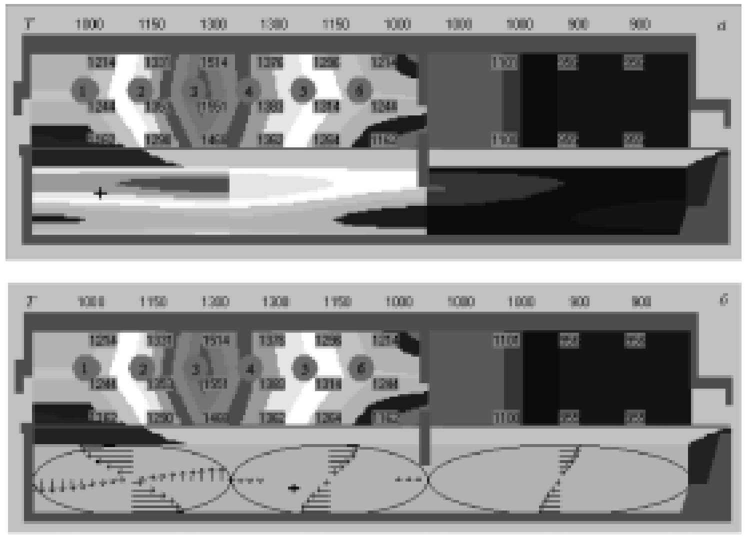

Fig. 2. Steady-state temperature field (a) and convective flows of the glass mass (b) in the longitudinal section of the glassmaking furnace (figures 1, 2, ..., 6 in the circles inside the furnace are the numbers of the burners; the remaining figures are the values of the temperatures in different zones of the furnace and of the ambient medium: time, within 20 h after the beginning of the processes; the temperature at the indicated + point (a) of the glass mass is 1350^{o}C; the horizontal (–15 m/h) and vertical (–2 m/h) components of the velocities at the indicated + point (b) of the glass mass.

The constructed mathematical model of dynamic thermal processes in a glassmaking furnace is qualitatively and quantitatively adequate to the real regime of melting of glass. As is shown by the investigations [3] and the preliminary comparison of the data of computer calculations and field measurements made under the conditions of glass melting in the actual furnace of the "Saratovsteklo" Public Corporation, the number of calculation points employed ensures accuracy of the calculation of the temperature fields and the fields of the velocities of glass-mass flow at a level of 10 to 20% in the nonstationary regimes of melting of glass and of units of percent in the steady-state regimes of melting of glass.

As the test calculation, we have modeled the regime of heating of a standard structure of a glassmaking furnace from the initial temperature T_0 = 1400^{o}C to the temperature of a prescribed working state of melting of glass.

Output of the visualized information on calculation of the temperature field and the convective glass-mass flows to the monitor of the computer is shown in Fig. 2.

The isotherms of the temperatures in the molten bath and in the zone of the furnace's gas medium are shown as contrasting shades of black and white (these pictures are colored on the screen of the monitor) in the scanning mode "Temperature Field" (Fig. 2a). The figures show the values of the temperatures at each instant of time.

The fields of the velocities of glass-mass flow in vector form are shown in the scanning mode "Convective Flows" (Fig. 2b).

Furthermore, at each instant of time, the user can point to any zone of the molten bath of the furnace with the mouse cursor and the values of the temperature and the components of the velocities of glass-mass flows in this zone will appear on the monitor. All the results of calculation of the temperature fields and velocity fields can be written in the protocol of the calculation.

The transient thermal and convective processes in different zones of the glassmaking furnace are given in Fig. 3.

As we see, changes in the temperature and the velocity of flow of the glass mass are the most intense in the melting zones below the central burners.

The amplitude of flow-velocity oscillations in the zone of melting of glass below the third burner attains ≈0.017 m/sec for a steady-state value of ≈0.0083 m/sec. The velocities of flow of the glass mass in the refining and cooling zones are approximately half as low. In the transition regime, the flow velocities have a damping oscillatory

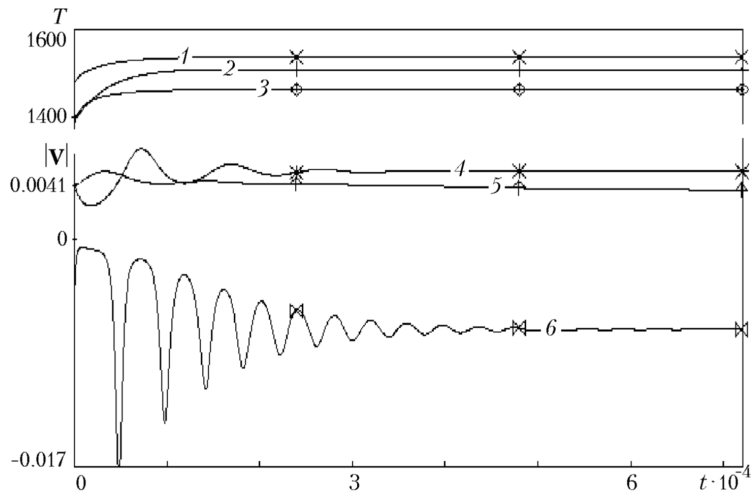

Fig. 3. Transient processes in the glassmaking furnace: 1, 2, and 3) maximum temperatures $T(t)$, ^{o}C, in the zones of the gas burners and of the furnace arc and the upper glass-mass layer; 4, 5 and 6) maximum horizontal velocities $|\mathbf{V}|$, m/sec, of the glass mass in the zones of refining, cooling, and melting.

character. The frequency of the oscillations of the velocity of glass-mass flow is approximately twice as high as the oscillation frequency in the refining zone.

Thus, the constructed mathematical model realized in the software system enables one to solve various practical problems of analysis of the heat and mass exchange in a glassmaking furnace and the synthesis problems on optimization of the parameters of the furnace, the characteristics of the glass mass, and control actions without carrying out series of expensive, energy-intensive, and lengthy field measurements.

NOTATION

∇, vector operator; $\overline{\text{grad}}$, gradient vector of the scalar function; ∇^2, Laplace operator; ρ, density of the glass mass, kg/m^3; $\mathbf{V}(x, z, t)$, vector field of flow velocities; $T(x, z, t)$, temperature field; $P(x, z, t)$, pressure field; $\mathbf{g}$, gravitational acceleration vector, m/sec^2; β, coefficient of thermal expansion of the glass mass, ^{o}C^{-1}; ν, kinematic viscosity of the glass mass, m^2/sec; k, thermal diffusivity of the glass mass, m^2/sec; t, time; $u(x, z, t)$ and $w(x, z, t)$, components of the velocity field along the x and z axes, m/sec; h, characteristic height of the glass-mass layer, m; T_0, nominal temperature, ^{o}C; ΔT, temperature difference, ^{o}C; $\theta(x, z, t)$, deviation from the z-linear profile of the temperature field, ^{o}C; $\psi(x, z, t)$, stream function, m^2/sec; $\psi_1(t)$, $\theta_1(t)$, and $\theta_2(t)$, amplitudes in the representations of the stream function, m^2/sec; λ, characteristic dimension of the closed trajectory of convective glass-mass flow in the x direction, m; r, φ, polar coordinates; F_φ, projection of the reduced mass forces onto the φ axis in the polar coordinate system, m/sec^2; $V_\varphi(z, \varphi)$, projection of the component of the flow-velocity vector in the polar coordinate system, m/sec; $P(\varphi)$, pressure, Pa; h_{ann} and R, geometric parameters (thickness and average radius) of the annular convective flows, m; $\Delta T(\varphi)$, configuration of the temperature field, ^{o}C; ΔT_0, maximum horizontal temperature difference, ^{o}C; μ, dynamic viscosity of the glass mass, Pa·sec; $z = r - R$, coordinate of the height of the molten glass-mass zone, m; $\langle V_\varphi \rangle$ and $\langle W_\varphi \rangle$, average components of the vector of the glass-mass-flow velocity, m/sec; p, correction factor; T_1, T_2, …, T_{31} and $\dot{T}_1$, $\dot{T}_2$, ..., $\dot{T}_{31}$, temperatures averaged over volume elements, ^{o}C, and their time derivatives, ^{o}C/sec; c_1, c_2, ..., c_{31}, effective heat capacities, J/ ^{o}C (the absence of subscripts from 19 to 25 is attributed to the fact that these number are "occupied" by the equations of motion of the glass mass in different zones of the furnace); $q_{1,2}$, $q_{1,7}$ ..., effective thermal conductivities between volume elements, W/^{o}C (the subscripts denote the numbers of the volumes between which we have heat exchange); $q_{7\text{med}}$, $q_{8\text{med}}$ ..., $q_{31\text{med}}$, thermal conductivities with the ambient medium, W/^{o}C (the figure in the subscript denotes the number of the volume element); p_1, p_2, and p_9, correction factors of the model (the presence of the omitted subscripts is due to the fact that in the paper there are no equations of the furnace cross section where they are employed); T_{med1}, T_{med2} ..., T_{med9}, average temperatures of the ambient medium in different zones around the furnace, ^{o}C; N_i and $N_i^{\max}$, strengths of the heat sources in a linear-relay-type thermal-control system, W (i

= 1, 2, ..., 6 corresponds to the number of the burner in the furnace); T_i. controlled temperature in the volume element, ^oC; T_i^{pr} and T_i^{lin}, parameters of the law of temperature control (prescribed temperature and zone of linear change of the temperature), ^oC. Subscripts and superscripts: ann, annular; med, medium; melt, melting; ref, refining; cool, cooling; max, maximum; pr, prescribed; lin, linear.

REFERENCES

1. D. B. Ginsburg, *Glassmaking Furnaces* [in Russian], Moscow (1967).
2. P. V. Levchenko, *Calculation of Furnaces and Dryers for the Silicate Industry* [in Russian], Moscow (1968).
3. V. É. Dzhashitov and V. M. Pankratov, *Dynamics of Temperature-Disturbed Gyroscopic Devices and Systems* [in Russian], Saratov (1998).
4. L. G. Loitsyanskii, *Mechanics of Liquids and Gases* [in Russian], Moscow (1987).
5. P. Berge, Y. Pomean, and C. Vidal, *Order in Chaos. The Deterministic Approach to Turbulence* [Russian translation], Moscow (1991).
6. H. G. Schuster, *Deterministic Chaos* [Russian translation], Moscow (1988).